Fixed-Parameter Tractability is Polynomial Time Extremal Structure Theory

Professor Mike Fellows
University of Newcastle, Australia

Most input distributions encountered in real-world applications of computing have highly significant extra structure that is not treated appropriately when only the overall instance size n is considered in complexity analysis and algorithm design.

Parameterized complexity unfolds from this empirical observation as essentially a two-dimensional generalization of the familiar one-dimensional framework of " P versus NP". The first part of the talk will cover the basics of this relatively new framework that pits *fixed-parameter tractability* (FPT) as the two-dimensional analog of P, against the two-dimensional analog of NP-hardness, which is hardness for $W[1]$. In this framework, the first dimension (as before) is the overall input size n, and the second dimension (the parameter) k provides a way of including the extra structure into problem complexity analysis and algorithm design. Some structural parameters, such as bounded treewidth, turn out to be nearly universal in practical scope.

The connection to extremal structure theory develops in the following way. One can show that a parameterized problem is in FPT if and only if an input instance (x, k) can be transformed in polynomial time (polynomial in both dimensions, n and k) into an "equivalent" instance (x', k') that satisfies:
(1) (x', k') is a yes-instance if and only if (x, k) is a yes instance,
(2) $|x'| \leq g(k)$ for some function $g(k)$, and
(3) $k' \leq k$.

In this way, FPT algorithm design inevitably generates the quite natural question of how powerful this *data-reduction* or *problem kernelization* can be. The transformation of (x, k) into the smaller equivalent instance (x', k') is essentially *pre-processing* in polynomial time, and as such is directly exportable to useful heuristics. (Powerful software packages for hard problems, such as CPLEX, often depend crucially on smart and muscular polynomial-time preprocessing.) In other words, we are interested in achieving $g(k)$ as small as possible in polynomial time. Thus FPT leads inevitably to polynomial-time extremal structure theory.

The talk is about a systematic approach to designing such extremal structure and kernelization algorithms. The history and recent achievements of this systematic approach will be surveyed, and the method will be illustrated by some new results on kernelization for the MAX LEAF SPANNING TREE problem.

(Joint work with V. Estivill-Castro, and F. Rosamond.)